Low-Complexity Iterative Sinusoidal Parameter Estimation

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Context

- Context: Approximating a signal as a sum of sinusoids
  - Audio compression
  - Audio processing
- Problem:
  - Estimating sinusoidal parameters is a non-linear problem
  - Non-linear problems are computationally expensive
  - Must often be done in real-time with few resources
- Solution
  - Linearising the problem as much as possible
  - Using an iterative solver

Sinusoidal Parameters

- A sinusoid is defined as
  - Amplitude
  - Phase
  - Frequency

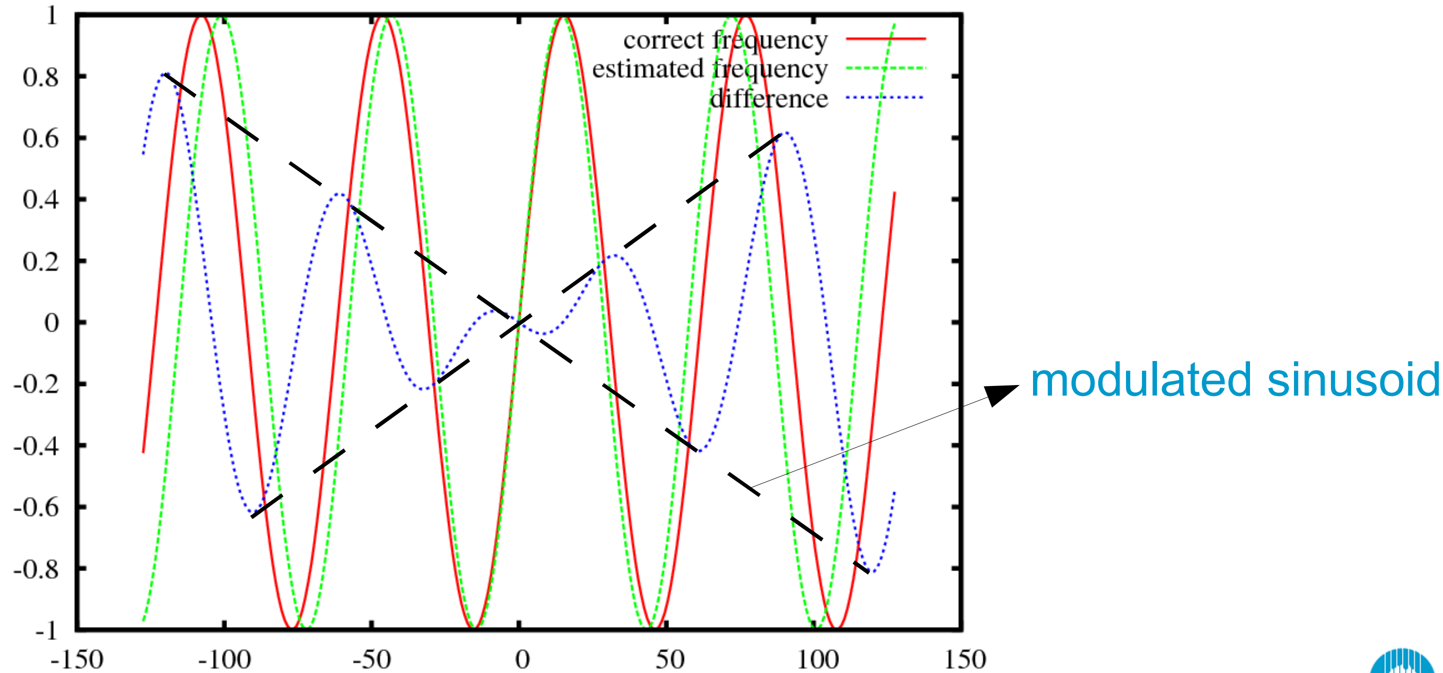
  (Amplitude and Phase:) Can be estimated linearly (e.g. FFT)

  (Frequency:) Non-linear
- We consider a fourth parameter
  - Linear amplitude modulation

$$\tilde{x}(n) = h(n) \sum_{k=1}^{N} \left(A_k + nA_k^{'} \right) \cdot \cos\left(\left(\theta_k + \Delta\theta_k \right) n + \phi_k \right)$$

Workaround: Linearisation

- Hypothesis #1: We have an initial estimate of frequencies
  - Obtained though a lower resolution FFT
  - From previous time frame
- Hypothesis #2: The error on the estimate is small
- Result: Frequency behaves *almost* linearly



Linear System

- Any sinusoid can be expressed as the sum of 4 basis functions

$$\tilde{x}(n) = h(n) \sum_{k=1}^{N} c_k \cos \theta_k n + s_k \sin \theta_k n + d_k n \cos \theta_k n + t_k n \sin \theta_k n$$

- Parameters are (neglecting 2nd order terms):

$$A_k = \sqrt{c_k^2 + s_k^2}$$

$$\phi_k = \arg\left(c_k - \jmath s_k\right)$$

$$A_k^{'} = \frac{d_k c_k + s_k t_k}{A_k}$$

$$\Delta\theta_k = \frac{d_k s_k - t_k c_k}{A_k^2}$$

Linear Solver

- Direct solver is $O(LN^2)$
- Iterative method: Gauss-Seidel in $O(LN)$
  - Basis is nearly orthogonal, guaranteed convergence
  - Successive projections of the error on the basis functions
  - First cos/sin terms, then modulated terms (faster convergence)

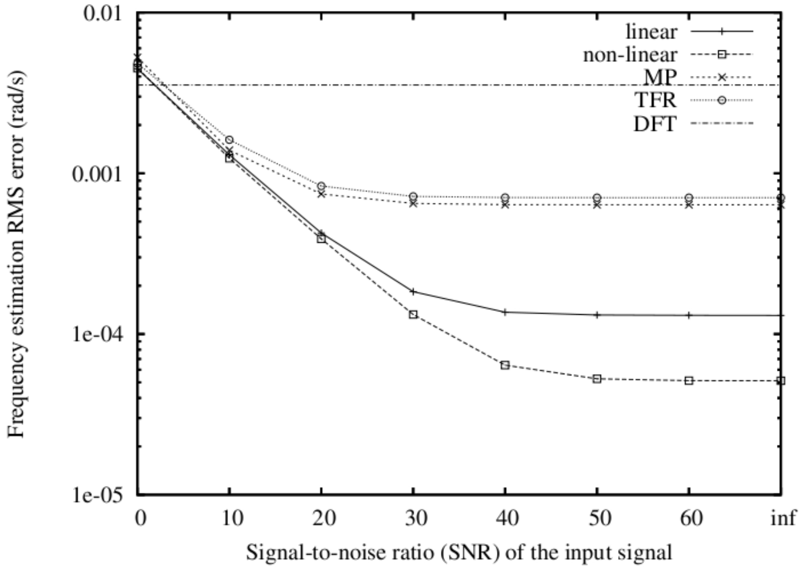
for all iteration i=1...M **do**
  for all sinusoid component $k = 1 \ldots 4N$ **do**
    $\Delta w_k^{(i)} \leftarrow \mathbf{a}_k^T \mathbf{e}$
    $\mathbf{e} \leftarrow \mathbf{e} - \mathbf{a}_k \Delta w_k^{(i)}$
    $w_k^{(i)} \leftarrow w_k^{(i-1)} + \Delta w_k^{(i)}$
  end for
end for

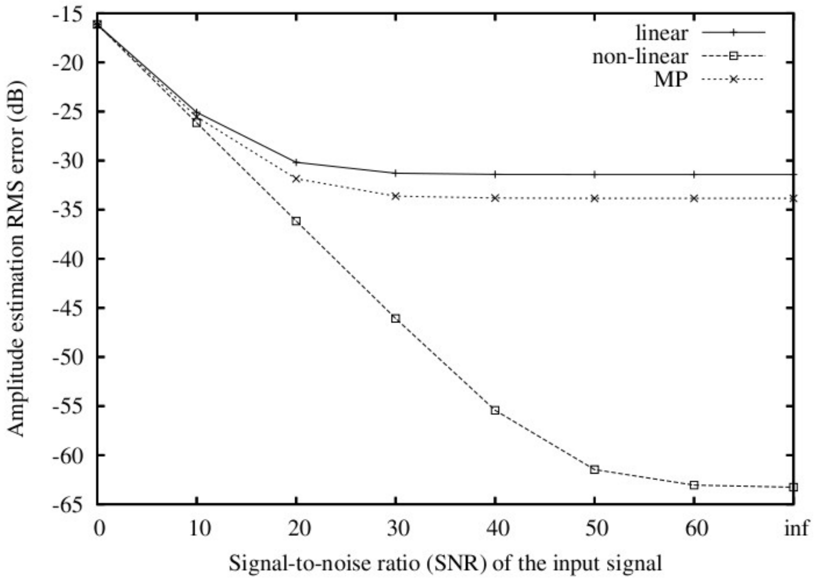
Non-Linear Solver

- Linear solution is imperfect when frequency error is too large
- Non-linear solver adjusts the frequency for every iteration
  1)Compute one linear iteration
  2)Compute sinusoid parameters (including new frequency)
  3)Recompute the error based on the non-linear parameters
  4)Goto 1)
- Complexity
  - Only a small increase compared to the linear solution:
    - Need to re-compute the basis functions
    - Slightly longer to converge

Results

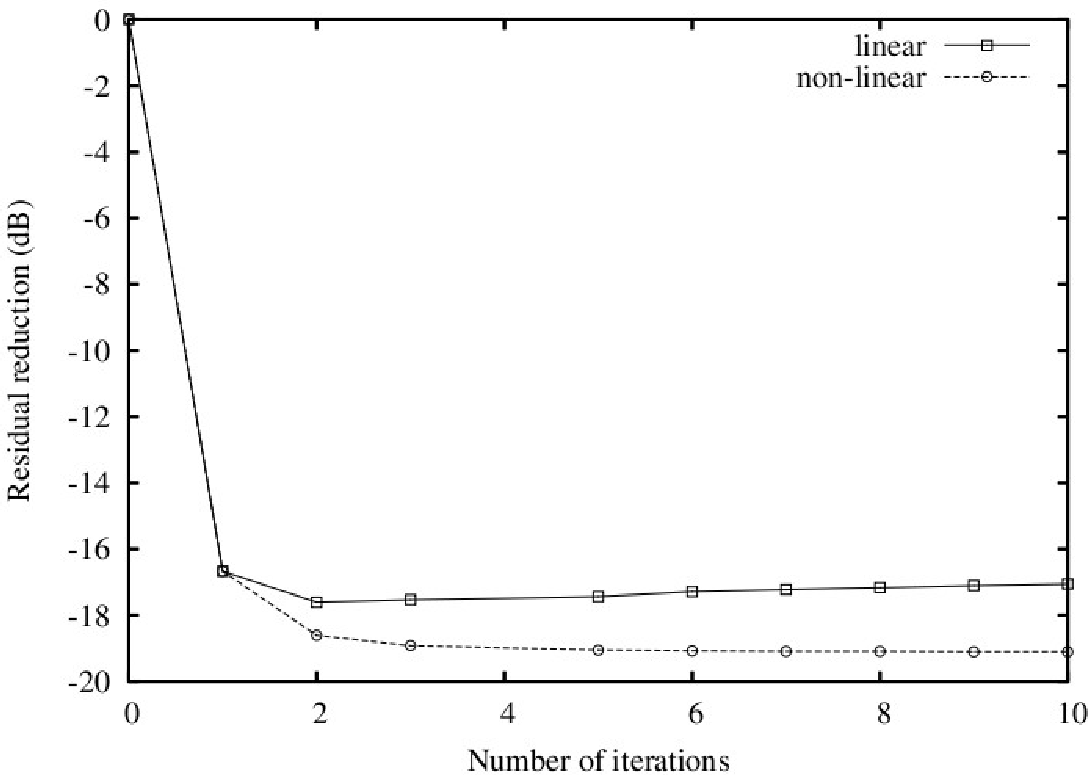
- Frequency and amplitude accuracy (5 chirps with noise)
  - Linear solution
  - Non-linear solution
  - Matching pursuit
  - Time-frequency reassignment
  - DFT





Convergence

- Convergence on a music signal
  - Linear solution requires 2 iterations
  - Non-linear solution requires 3 iterations



Complexity

- L: Length of the input data (256)
- M: Number of iterations (2 for linear, 3 for non-linear)
- N: Number of sinusoids (20)
- P: Matching pursuit oversampling (32)

Algorithm	Complexity	Typical (Mflops)
MP (slow)	$4LN^2P$	3,300
MP (FFT)	$\frac{5}{2}LNP\log_2 LP$	1,300
linear (18)	$64N^3 + 32LN^2$	900
non-linear ([5])	$O\left(N^4 + LN^2\right)$	>500*
linear (proposed)	$(8M + 5)\,LN$	27
non-linear (prop.)	$(17M - 4)\,LN$	60

Conclusion

- A low-complexity method for estimating sinusoid parameters
  - Linearisation of the estimation problem
  - Iterative solution (Gauss-Seidel)
  - Optional non-linear solution
- Reduces complexity by 1-2 orders of magnitude compared to other algorithms
- Future work
  - Improve initial frequency estimates
  - Extend to the estimation of frequency modulation

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